

AD-A049 420

WISCONSIN UNIV MADISON MATHEMATICS RESEARCH CENTER  
A BIBLIOGRAPHY OF FREE BOUNDARY PROBLEMS.(U)

F/G 12/1

SEP 77 C W CRYER  
MRC-TSR-1793

DAA629-75-C-0024  
NL

UNCLASSIFIED

1 OF 1  
AD  
A049420



END  
DATE  
FILMED  
3-78

DDC

13

AD A049420

FILE COPY

MRC Technical Summary Report # 1793

A BIBLIOGRAPHY OF FREE BOUNDARY  
PROBLEMS

Colin W. Cryer

Mathematics Research Center  
University of Wisconsin-Madison  
610 Walnut Street  
Madison, Wisconsin 53706

September 1977

(Received July 11, 1977)

See back  
page for 1473

DDC  
RECEIVED  
FEB 2 1978  
D

Approved for public release  
Distribution unlimited

Sponsored by

U.S. Army Research Office  
P. O. Box 12211  
Research Triangle Park  
North Carolina 27709

and

National Science Foundation  
Washington, D. C.  
20550

ADDITIONAL	
NTIS	White Section <input checked="" type="checkbox"/>
DDC	Dark Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
DISC	AVAIL. NO. OF SPECIAL
A	

Microfiche removed, not  
AMA Standard; DDC cannot  
reproduce.

UNIVERSITY OF WISCONSIN - MADISON  
MATHEMATICS RESEARCH CENTER

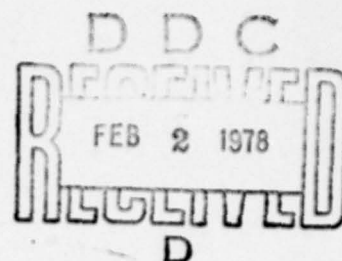
A BIBLIOGRAPHY OF FREE BOUNDARY PROBLEMS

Colin W. Cryer

Technical Summary Report #1793

September 1977

ABSTRACT



A free boundary problem is a (steady-state) boundary value problem involving differential equations on domains parts of whose boundaries, the free boundaries, are unknown and must be determined as part of the solution. Free boundary problems occur in all branches of continuum mechanics. A typical free boundary problem is the problem of a water jet in air, where the unknown free boundary is the water/air interface.

The bibliography contains about 3300 references, most of which are classified according to one or more of approximately 1200 subjects. This report contains a description of the bibliography and a list of the subjects, which are divided into three main groups: I, Types of free boundary problems; II, Mathematical methods; and III, Approximation methods. Listings by author and subject are given in appendices; these appendices are on microfiche, but paper copies may be obtained by writing to the Mathematics Research Center.

AMS(MOS) Subject Classification - 31-02, 35J99, 35R99, 45-02, 49-02, 65-02,  
65P05, 65R05, 73-02, 76-02, 78-02, 80-02, 85-02, 86-02.

Key Words - Free boundary problems, Fluid mechanics, Porous flow, Mechanics of solids, Heat conduction and diffusion, Electromagnetism, Gravitation, Mathematical methods, Numerical methods.

Work Unit Number 3 - Applications of Mathematics

## A BIBLIOGRAPHY OF FREE BOUNDARY PROBLEMS

Colin W. Cryer

1. Description of the bibliography. A free boundary problem (FBP; plural, FBPS) is a (steady-state) boundary value problem involving differential equations on domains parts of whose boundaries, the free boundaries (FB; plural FBS), are unknown and must be determined as part of the solution.

FBPS arise in a great variety of contexts, and the literature on the subject is scattered throughout many different disciplines. The present bibliography has been compiled in an attempt to bring together all the important results on FBPS. It is our hope that this survey will reduce the duplication of effort and stimulate the cross-fertilization of ideas among the different disciplines concerned with FBPS.

The bibliography contains about 3300 references most of which are classified according to one or more of approximately 1200 subjects.

In the remainder of this section we make some general remarks about the bibliography. In section 2 we describe the approach used to classify the different types of FBPS in continuum mechanics. In section 3 we give abbreviations and conventions. Finally, section 4 contains a list of the subjects. Listings by author and by subject are given in appendices; these appendices are on microfiche, but paper copies may be obtained by writing to the Mathematics Research Center.

References for the bibliography were obtained in the following ways:

- (i) By searching the recent issues of about forty journals which contain a relatively large number of papers on FBPS. The following journals contained an especially large proportion of papers on FBPS:

Archive for Rational Mechanics and Analysis  
Communications on Pure and Applied Mathematics  
Journal of Fluid Mechanics  
Physics of Fluids

---

Sponsored by the United States Army under Contract No. DAAG29-75-C-0024 and by the National Science Foundation under Grant No. DCR75-03838.



Proceedings American Society of Civil Engineers (Division of Engineering Mechanics, Division of Hydraulics and Division of Soil Mechanics)

Water Resources Research

- (ii) By following up the bibliographies of books and papers.
- (iii) By writing to a large number of authors requesting reprints of their recent work.
- (iv) By using the following abstracting services:
  - Applied Mechanics Reviews
  - Computing Reviews
  - Contents of Contemporary Mathematical Journals
  - Dissertation Abstracts International
  - Mathematical Reviews
  - Science Citation Index
- (v) By browsing in libraries and bookstores, and chatting to colleagues.

Almost every reference was consulted and copies were obtained of almost all papers and many of the books.

The literature on FBPS is enormous. To quote a typical example, between 1965 and 1968 over five hundred papers were published on the motion of bubbles and drops. In consequence, we had to be selective in deciding which references to include. The following criteria were used in determining which references should be included:

- (i) Papers describing heuristic approximation methods, papers describing experimental results, and papers using the well-known hodograph method are not usually included.
- (ii) All papers known to us giving existence or uniqueness theorems are included.
- (iii) All papers known to us describing numerical methods are included.
- (iv) Whenever possible, a worker in the field is represented by at least one paper.
- (v) When an author has himself presented a summary of his results, perhaps in the form of a book, sometimes only the summary and particularly noteworthy papers are included.

- (vi) When a series of papers forms a natural sequence - perhaps giving first-order, second-order, and higher-order approximations - often only the last paper in the sequence is included.
- (vii) Each approach and each type of problem known to us is represented by at least one reference.
- (viii) Papers containing interesting new ideas are included even when not directly concerned with FBPS when it is felt that the results have possible future applications to FBPS.
- (ix) Preference is usually given to recent work, but papers of historical interest are included.

The over-riding criterium in selecting references was that the bibliography should remain comprehensible and manageable to us and useful to the reader.

In constructing the list of subjects, our goal was to devise a systematic classification which was so organized that the information could be readily retrieved. The classification which we use is rather formidable, but it is hoped that it will make it possible for the reader interested in a particular problem to locate precisely the information of interest to him. There are three broad classes of subjects: I, Types of free boundary problems; II, Mathematical methods; and III, Approximation methods. The classification of FBPS in continuum mechanics presented especial difficulties which are discussed in section 2. Most references of the bibliography are followed by a list of cross-references to the subjects with which they are concerned.

Of the major scientific languages other than English, we read German fluently and read French, Italian, and Russian with difficulty. This has affected the choice of references to some extent, and has obviously affected our ability to comprehend certain references, particularly long references in Russian; such references are therefore cross-referenced less frequently than they deserve.

The bibliography has been constructed in conjunction with a series of surveys on FBPS which are appearing as Mathematics Research Center Technical Summary Reports. The following comments arise:

(i) Two surveys have already appeared, namely Technical Summary Report #1657 on porous flow FBPS which corresponds to I.4 in the subject listing, and Technical Summary Report #1693 on trial free-boundary methods, which corresponds to III.3 in the subject listing. References quoted in these two surveys are listed in the present bibliography, and some new references have also been added.

(ii) Many sections of the other surveys have already been written. Some of the terminology used in the list of subjects and some of the cross-references made will perhaps seem obscure to the reader and may only become clear when the corresponding surveys appear.

The bibliography was begun during a sabbatical spent in 1972 at Oxford University with the support of the United Kingdom Science Research Council and the National Science Foundation, and would not have been written had it not been for the interest in FBPS of Professor L. Fox of the Oxford University Computing Laboratory.

The National Science Foundation (through Contract No.: GJ-33096) provided substantial help which enabled us to work full-time on the project for nine months and to meet the costs of xeroxing and typing. We are particularly grateful in that additional support was given when the original estimates of time and cost proved to be hopelessly wrong.

Thanks are due to the staff of the Radcliffe Science Library (Oxford), and the Memorial, Physics, and Engineering Libraries (Madison), for their help and forbearance.

We would welcome information about errors and omissions, as well as reprints of work on FBPS. These would be incorporated in a revised edition of this bibliography which we hope to prepare.



2. Classification of FBPS in continuum mechanics. There is no completely satisfactory method of classifying FBPS in continuum mechanics. The approach followed is:

- (i) To classify first according to the broad subdivisions of continuum mechanics such as fluid mechanics.

The classification is based upon the "nontrivial" field equations occurring. We illustrate this by means of some examples. The problem of water waves subject to gravity involves both fluid mechanics and gravitation, but the gravitational field is given so that the problem is classified under fluid mechanics. The problem of a rotating self-gravitating fluid mass also involves both fluid mechanics and gravitation, but in this case the fluid velocity is usually prescribed, so that the problem is classified under gravitation. Problems involving a plasma in equilibrium under a magnetic field are often described as magnetohydrodynamic problems but in many cases the hydrodynamic equations are trivially satisfied and in such cases the problems are classified under electromagnetism.

Problems involving two nontrivial fields are treated in one of two ways:

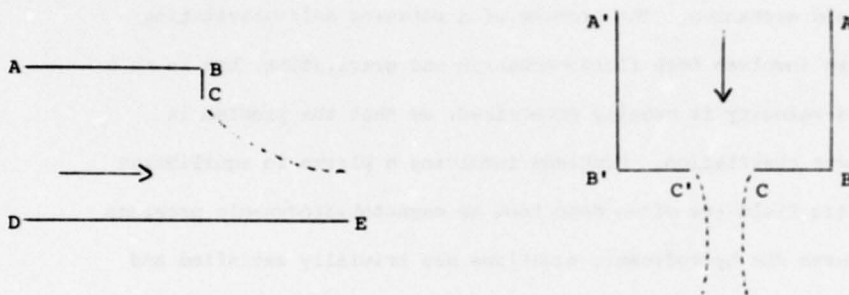
- (a) If the problem can be regarded as a generalization of a FBP in one area of continuum mechanics (the primary area), then the problem is classified under the primary area. For example, porous flow FBPS can be generalized by considering electrical effects: the resulting electrokinetic porous flow problems are classified under porous flow problems.
- (b) If a problem inherently involves two or more areas of continuum mechanics, then it is classified under coupled field problems. For example, the flow of a blood corpuscle through a capillary is quite different from problems in either fluid dynamics or elasticity and is classified as an elasto-hydrodynamic problem in the chapter on coupled field problems.



When a problem is a combination of two FBPS, it is classified according to the primary problem. For example, the problem of a cavitating hydrofoil under a water surface is classified under cavities because this is of primary interest.

(ii) To classify next according to the geometry of the problem.

A major difficulty is that by interchanging axes or invoking symmetry a problem can be transformed into an apparently different problem. Consider, for example, the horizontal flow of a fluid out of an obstructed channel (Figure 1a).



(a) Horizontal flow from a channel

(b) Vertical jet flow

Figure 1: Equivalent problems

When gravity is neglected the boundary DE is an axis of symmetry and the problem is equivalent to a vertical jet (Figure 1b). However, when gravity is taken into account or when the fixed boundaries are slightly changed, the two problems are no longer equivalent.

Axisymmetric problems are classified according to their plane equivalents. For example, the plane equivalent of an axisymmetric is a plane jet, but the plane equivalent of an axisymmetric stream flowing around a wire is a stream flowing down a plane wall. This approach is quite appropriate because of the close relationship between corresponding plane and axially symmetric problems.

(iii) To classify finally according to the governing equations and boundary conditions.

For example, the vertical jet shown in Figure 1b is considered for various governing equations (inviscid flow, viscous flow, etc.) and various boundary conditions (no gravity, gravity, surface tension, etc.)

The method of classification described above (like any other) cannot eliminate all anomalies. The same mathematical problem may arise in two different areas of continuum mechanics. Should the problem of a hydrofoil cavity underneath a water surface be classified under cavities or under water waves? Does a problem have a symmetrical equivalent? The reader must be prepared to look for a problem under more than one heading.

### 3. Abbreviations and conventions.

The bibliography is arranged alphabetically. A prefix is treated as part of the name so that von Mises is listed under V and not under M. Names beginning with Mac and Mc are listed under MAC and MC, respectively.

The transliteration of Cyrillic names is handled as follows. If a reference has been translated, then the spelling of the translation is used. If a reference has not been translated then a review is quoted and the spelling of the review is used. In some cases this approach leads to an author's name appearing with different spellings. When this would appear to create misunderstanding, alternative spellings are given in brackets. (For many years we were unaware that the famous Joukowsky in fluid mechanics was the same person as the famous Zhukovsky in porous flow.)

When a reference has appeared in two essentially equivalent forms this is indicated by means of an "=" ; the date used is usually the earliest date.

Proceedings and Symposia are listed under the editors when known. Otherwise, they are listed, not very systematically, under "Proceedings", "Symposium", or the conference title.

When a reference has been reported to contain errors, the source of this information is indicated. Occasionally, general observations about a reference are also included.

In general, each reference is followed by a list of cross-references to the subjects to which it is relevant. Some very important references are referred to so often that the list of cross-references is very long. In such cases the list of references is given in the author listing but is omitted in the subject listings.

As mentioned in section 1, the subjects listed under I.4 (porous flow problems) and III.3 (trial free-boundary methods) correspond to two published surveys but also include some new references. These "new" references are identified by an "N" following the cross-reference, and in the subject listing they are listed separately following the "old" references.

The following abbreviations are used:

U : Unknown or Unclassified  
TA, TAA, etc.: To appear, to appear (a), etc.  
CR : Computing Reviews  
DA : Dissertation Abstracts International  
MR : Mathematical Reviews  
N : New references  
\*\* : List of cross-references omitted in subject listing.

Theses which are listed in Dissertation Abstracts are followed by their order number.



4. Author Index and Subject Index

AUTHOR INDEX			
AUTHOR	APPENDIX	PAGE	* FICHE
A	1	1	B 1
B	1	8	I 1
C	1	27	B 3
D	1	42	D 4
E	1	50	L 4
F	1	53	B 5
G	1	61	J 5
H	1	75	K 6
I	1	86	I 7
J	1	87	J 7
K	1	91	N 7
L	1	100	J 8
M	1	111	H 9
N	1	129	M10
O	1	133	D11
P	1	135	F11
Q	1	142	M11
R	1	142	M11
S	1	148	F12
T	1	167	L13
U	1	176	H14
V	1	176	H14
W	1	180	L14
Y	1	187	F15
Z	1	190	I15

\* Microfiche removed, not AMA Standard, DDC cannot reproduce

# SUBJECT INDEX

SUBJECT	APPENDIX	PAGE	FICHE
I	2	1	B 1
I.1	2	1	B 1
I.1.1	2	1	B 1
I.1.2	2	1	B 1
I.1.3	2	1	B 1
I.2	2	1	B 1
I.2.1	2	2	C 1
I.2.1.1	2	2	C 1
I.2.1.2	2	2	C 1
I.2.1.3	2	2	C 1
I.2.2	2	2	C 1
I.2.2.1	2	2	C 1
I.2.3	2	3	D 1
I.2.4	2	3	D 1
I.2.4.1	2	3	D 1
I.2.4.1.1	2	3	D 1
I.2.4.1.2	2	4	E 1
I.2.4.1.3	2	4	E 1
I.2.4.1.4	2	5	F 1
I.2.4.1.5	2	5	F 1
I.2.4.2	2	5	F 1
I.2.4.2.1	2	5	F 1
I.2.4.2.2	2	5	F 1
I.2.4.2.3	2	5	F 1
I.2.4.3	2	5	F 1
I.2.4.4	2	5	F 1
I.3	2	5	F 1
I.3.1	2	5	F 1
I.3.1.1	2	7	M 1
I.3.1.2	2	8	I 1
I.3.1.3	2	8	I 1
I.3.1.3.1	2	8	I 1
I.3.1.3.1.1	2	9	J 1
I.3.1.3.1.2	2	9	J 1
I.3.1.3.1.3	2	9	J 1
I.3.1.3.2	2	9	J 1
I.3.1.3.2.1	2	9	J 1
I.3.1.3.2.2	2	9	J 1
I.3.1.3.3	2	9	J 1
I.3.1.3.3.1	2	10	K 1
I.3.1.3.3.2	2	10	K 1
I.3.1.3.3.3	2	10	K 1
I.3.1.3.4	2	10	K 1
I.3.1.3.4.1	2	10	K 1
I.3.1.3.4.2	2	11	L 1
I.3.1.3.4.3	2	11	L 1
I.3.1.3.5	2	11	L 1
I.3.1.3.5.1	2	11	L 1
I.3.1.3.5.2	2	12	M 1
I.3.1.3.5.3	2	12	M 1
I.3.1.3.5.4	2	13	N 1
I.3.1.3.5.5	2	13	N 1
I.3.1.3.5.6	2	13	N 1
PART I. TYPES OF FREE BOUNDARY PROBLEMS			
INTRODUCTION			
FORMULATION OF FBPS			
ORIGINS OF FBPS			
CLASSIFICATION OF FBPS			
PRINCIPLES OF CONTINUUM MECHANICS			
MATHEMATICAL PRELIMINARIES			
VECTORS AND TENSORS			
VECTORS AND TENSOR FIELDS			
SPECIFIC CO-ORDINATE SYSTEMS			
BASIC PRINCIPLES			
THE MOTION OF BODIES			
SINGULAR SURFACES			
APPROXIMATIONS IN CONTINUUM MECHANICS			
INVOLVING ALL TYPES OF PROBLEMS			
UNCOUPLLED FIELDS			
SCALING			
CONSTITUTIVE EQUATIONS			
REDUCTION OF DIMENSION			
BOUNDARY CONDITIONS			
INVOLVING FBPS			
ELIMINATION OF FBPS			
INTERFACIAL EFFECTS			
ARTIFICIAL FBPS			
AN EXAMPLE			
CONCLUDING REMARKS			
FLUID MECHANICS			
THE GOVERNING EQUATIONS			
IRROTATIONAL INCOMPRESSIBLE INVISCID FLOW			
THE GENERAL EQUATIONS			
THE CONSTITUTIVE EQUATION			
COMPRESSIBLE INVISCID FLOW			
CHAPLYGIN GAS			
KARMAN-TSIEN TANGENT GAS			
SHOCK WAVES			
VISCOUS NEWTONIAN FLUIDS			
STOKES FLOW			
HELE - SHAW FLOW			
NON-NEWTONIAN FLUIDS			
BINGHAM FLUIDS			
VISCOELASTIC FLUIDS			
MOLECULAR FLUIDS			
LIQUID CRYSTALS			
SUPERFLUIDS			
FERROFLUIDS			
SPECIAL REGIMES			
BOUNDARY LAYER			
LUBRICATION			
TURBULENCE			
CAVITATION			
STRATIFIED FLOW AND GASKINETIC FLOW			
INVISCID INCOMPRESSIBLE WITH VORTICITY			

SUBJECT	APPENDIX	PAGE	FICHE
I.3.1.3.5.7		13	N 1
I.3.1.3.5.8		13	N 1
I.3.1.3.5.9		14	B 2
I.3.1.3.6		14	B 2
I.3.2		15	C 2
I.3.2.1		15	C 2
I.3.2.1.1		15	C 2
I.3.2.1.2		15	C 2
I.3.2.1.3		15	C 2
I.3.2.2		15	C 2
I.3.2.2.1		15	C 2
I.3.2.2.1.1		16	D 2
I.3.2.2.1.2		16	D 2
I.3.2.2.1.3		16	D 2
I.3.2.2.2		16	D 2
I.3.2.2.2.1		17	E 2
I.3.2.2.2.2		17	E 2
I.3.2.2.2.3		18	F 2
I.3.2.2.2.4		18	F 2
I.3.2.2.2.5		19	G 2
I.3.2.2.3		19	G 2
I.3.3		20	H 2
I.3.3.1		20	H 2
I.3.3.2		20	H 2
I.3.3.2.1		20	H 2
I.3.3.2.1.1		21	I 2
I.3.3.2.1.2		22	J 2
I.3.3.2.1.2.1		22	J 2
I.3.3.2.1.2.2		22	J 2
I.3.3.2.1.2.3		23	K 2
I.3.3.2.1.3		23	K 2
I.3.3.2.1.3.1		23	K 2
I.3.3.2.1.3.2		23	K 2
I.3.3.2.1.3.3		23	K 2
I.3.3.2.1.4		23	K 2
I.3.3.2.1.4.1		23	K 2
I.3.3.2.1.4.2		23	K 2
I.3.3.2.1.4.3		24	L 2
I.3.3.2.1.5		24	L 2
I.3.3.2.1.5.1		24	L 2
I.3.3.2.1.5.2		24	L 2
I.3.3.2.2		24	L 2
I.3.3.2.2.1		25	M 2
I.3.3.2.2.2		25	M 2
I.3.3.2.2.3		26	N 2
I.3.3.2.2.3.1		26	N 2
I.3.3.2.2.3.2		26	N 2
I.3.3.2.2.3.3		26	N 2
I.3.3.2.2.4		27	B 3
I.3.3.2.2.5		27	B 3
I.3.3.2.3		27	B 3
THIN FILMS			
BORES			
MULTIPHASE FLOW			
BOUNDARY CONDITIONS			
FIXED BOUNDARIES			
RIGID			
POROUS			
AT INFINITY			
FREE BOUNDARIES			
CONTINUITY OF STRESS			
INVISCID			
VISCOUS			
FLEXIBLE BOUNDARIES			
INTERFACIAL EFFECTS			
CONSTANT SURFACE TENSION			
VARIABLE SURFACE TENSION			
TEMPERATURE AND CONCENTRATION DEPENDENCE			
SURFACE CONTAMINATION			
HYSTERESIS			
MECHANICAL INTERFACES			
DIFFUSE INTERFACES			
NEW SURFACES			
POINTS OF DETACHMENT			
SUBSONIC CAVITIES AND WAKES			
EXPERIMENTALLY OBSERVED PROPERTIES			
CAVITY MODELS			
INFINITE WAKE MODELS			
KIRCHHOFF-RAYLEIGH MODEL			
OPEN-WAKE MODELS			
JOUKOWSKI-ROSHKO-EPLER			
WORDS			
WU			
VORTEX MODELS			
WEINIG			
TULIN SINGLE-VORTEX			
TULIN DOUBLE-VORTEX			
AEROFOIL MODELS			
SCHNIEDEN			
SPENCE			
HELICOPTER BLADES			
MODELS WITH SINGULARITIES			
HOPKINSON			
PARKINSON-JANDALI			
FINITE WAKE MODELS			
CUSPED WAKE MODEL			
RIABOUCHINSKY MODEL			
MODIFIED			
RECIRCULATING MODELS			
FOEPPL			
BATCHelor			
LAVRENTIEV			
RE-ENTRANT MODEL			
MODELS WITH ACCELERATION, GRAVITY, AND SURFACE TENSION			
MISCELLANEOUS MODELS			

SUBJECT	APPENDIX	PAGE	FIGURE
1.3.3.2.3.1	2	27	B 1
1.3.3.2.3.2	2	28	C 1
1.3.3.3	2	29	D 1
1.3.3.4	2	30	E 1
1.3.3.4.1	2	30	E 1
1.3.3.4.1.1	2	30	E 1
1.3.3.4.1.2	2	31	F 1
1.3.3.4.2	2	32	G 1
1.3.3.4.2.1	2	32	G 1
1.3.3.4.2.2	2	33	H 1
1.3.3.4.3	2	33	H 1
1.3.3.4.3.1	2	33	H 1
1.3.3.4.3.2	2	33	H 1
1.3.3.4.4	2	33	H 1
1.3.3.4.4.1	2	33	H 1
1.3.3.4.4.2	2	33	H 1
1.3.3.4.5	2	35	J 1
1.3.3.4.6	2	35	J 1
1.3.3.4.7	2	36	K 1
1.3.3.4.8	2	38	M 1
1.3.4	2	36	M 1
1.3.4.1	2	39	N 1
1.3.4.1.1	2	39	N 1
1.3.4.1.1.1	2	39	N 1
1.3.4.1.1.1.1	2	39	N 1
1.3.4.1.1.1.2	2	39	N 1
1.3.4.1.1.2	2	41	C 1
1.3.4.1.1.2.1	2	41	C 1
1.3.4.1.1.2.2	2	41	C 1
1.3.4.1.1.2.3	2	41	C 1
1.3.4.1.1.3	2	41	C 1
1.3.4.1.1.4	2	41	C 1
1.3.4.1.2	2	41	C 1
1.3.4.1.3	2	42	D 1
1.3.4.1.3.1	2	43	E 1
1.3.4.1.3.2	2	43	E 1
1.3.4.1.3.3	2	43	E 1
1.3.4.1.3.4	2	43	E 1
1.3.4.1.3.5	2	45	F 1
1.3.4.2	2	46	M 1
1.3.4.2.1	2	47	I 1
1.3.4.2.1.1	2	48	J 1
1.3.4.2.1.1.1	2	48	J 1
1.3.4.2.1.1.2	2	48	J 1
1.3.4.2.1.2	2	48	J 1
1.3.4.2.1.3	2	49	K 1
1.3.4.2.2	2	49	K 1
1.3.4.3	2	50	L 1
BOUNDARY LAYER			
NAVIER STOKES			
COMPARISON OF THE MODELS			
SPECIFIC GEOMETRIES			
FLAT PLATE OBSTACLES			
PLANE			
AXISYMMETRIC(DISC)			
WEDGE-SHAPED OBSTACLES			
WEDGE			
CONE			
CIRCULAR ARC OBSTACLES			
PLANE			
AXISYMMETRIC			
CIRCULAR OBSTACLES			
CYLINDER			
SPHERE			
ELLIPSOIDAL OBSTACLES			
GENERAL SYMMETRIC OBSTACLES			
GENERAL UNSYMMETRIC OBSTACLES			
HYDROFOILS			
JETS (TWO FREE SURFACES)			
ORIFICE JETS			
WALL JETS			
INVISCID, INCOMPRESSIBLE			
NO GRAVITY OR SURFACE TENSION			
PLANE			
AXISYMMETRIC			
GRAVITY			
PLANE			
AXISYMMETRIC			
THREE DIMENSIONAL			
CAPILLARY			
GRAVITY/CAPILLARY			
INVISCID COMPRESSIBLE			
GENERAL GOVERNING EQUATIONS			
JETS FROM TUBES			
SYMMETRIC FLOWS (VERTICAL)			
CAPILLARY			
NEWTONIAN (VISCOUS)			
GRAVITY/CAPILLARY			
GENERAL CONSTITUTIVE EQUATIONS			
UNSYMMETRIC FLOWS (HORIZONTAL)			
POLYGONAL ORIFICES			
CURVED ORIFICES			
GENERAL GEOMETRY			
WEIRS			
SHARP-CRESTED WEIRS			
INFINITELY HIGH			
PLANE			
AXIALLY SYMMETRIC			
WATERFALLS			
GENERAL CASE			
OTHER PROBLEMS			
IMPINGING JETS			



SUBJECT	APPENDIX	PAGE	FICHE
1.3.4.3.1	2	50	L 4
1.3.4.3.2	2	51	M 4
1.3.4.3.3	2	51	M 4
1.3.4.3.4	2	51	M 4
1.3.4.3.5	2	52	N 4
1.3.4.3.6	2	53	N 4
1.3.4.4.1	2	53	H 5
1.3.4.4.2	2	53	H 5
1.3.5	2	53	H 5
1.3.5.1	2	54	C 5
1.3.5.1.1	2	54	C 5
1.3.5.1.2	2	55	D 5
1.3.5.2	2	55	D 5
1.3.5.2.1	2	55	D 5
1.3.5.2.2	2	55	D 5
1.3.5.2.3	2	55	D 5
1.3.5.3	2	56	E 5
1.3.5.4	2	56	E 5
1.3.5.5	2	57	F 5
1.3.5.5.1	2	57	F 5
1.3.5.5.2	2	57	F 5
1.3.5.5.3	2	57	F 5
1.3.5.5.4	2	57	F 5
1.3.6	2	57	F 5
1.3.6.1	2	58	G 5
1.3.6.2	2	58	G 5
1.3.6.3	2	59	H 5
1.3.6.4	2	60	I 5
1.3.6.5	2	60	I 5
1.3.6.6	2	60	I 5
1.3.7	2	61	J 5
1.3.7.1	2	61	J 5
1.3.7.1.1	2	62	K 5
1.3.7.1.2	2	62	K 5
1.3.7.1.3	2	63	L 5
1.3.7.1.4	2	64	M 5
1.3.7.1.5	2	65	N 5
1.3.7.1.6	2	65	N 5
1.3.7.1.7	2	66	F 5
1.3.7.1.8	2	67	C 5
1.3.7.1.9	2	67	C 5
1.3.7.1.10	2	67	C 5
1.3.7.1.11	2	68	D 5
1.3.7.2	2	68	D 5
1.3.7.2.1	2	68	D 5
1.3.7.2.1.1	2	68	D 5
1.3.7.2.1.2	2	69	E 5
1.3.7.2.2	2	69	E 5
1.3.7.2.2.1	2	69	E 5
1.3.7.2.2.1.1	2	69	E 5
1.3.7.2.2.1.1.1	2	69	E 5
1.3.7.2.2.1.1.2	2	70	F 5
1.3.7.2.2.1.2	2	70	F 5
1.3.7.2.2.2	2	71	G 5
AGAINST A HALF-PLANE			
AGAINST OTHER JETS			
AGAINST A FLUID SURFACE			
PENETRATING			
AGAINST OBSTACLES			
SURMERGED JETS			
IMPINGING ON A FREE SURFACE			
INJECTED INTO A STRAM			
FLAWS WITH ONE UNOBSTRUCTED SURFACE			
SURMERGED OBSTACLES			
PLANE			
AXISYMMETRIC			
SPILLWAYS			
CREST			
FACE			
TOE			
SURMERGED VORTICES			
CHANNEL FLOWS			
VISCOUS WAVES ON SLOPES			
MONOCLINAL			
POLYCLINAL			
SOLITARY			
ROLL			
FLAWS WITH ONE UNOBSTRUCTED SURFACE			
PLANING SURFACES			
CONDUITS			
UNDERFLOW GATES (SLUTCF GATES)			
COMPOSITE HYDRAULIC FLOWS			
TRAILING EDGES			
OTHER FLOWS			
BUBBLES AND DROPS			
SINGLE BUBBLE IN UNCONFINED FLOW			
SPHERICAL			
SPHEROIDAL			
INDENTED SPHEROIDAL			
FALLING DROPS			
POINTED SPHEROIDAL			
SPHERICAL CAP			
SPHERICAL CAP WITH SKIRTS			
SPHERICAL CAP WITH FUNNEL			
TOROIDAL			
SPIRALLING OF 7IG-ZAGGING			
OSCILLATING			
SINGLE BUBBLE IN CONFINED FLOW			
FINITE BUBBLES			
SYMMETRIC			
UNSYMMETRIC			
INFINITE BUBBLES			
SYMMETRIC			
INVISCID			
PLANE			
AXISYMMETRIC			
VISCOUS			
UNSYMMETRIC			

SUBJECT	APPENDIX	PAGE	FICHE
1.3.7.3	2	71	G 6
1.3.7.3.1	2	71	G 6
1.3.7.3.2	2	72	H 4
1.3.7.3.3	2	72	H 4
1.3.7.4	2	72	H 4
1.3.7.4.1	2	72	H 4
1.3.7.4.2	2	72	H 4
1.3.7.4.3	2	72	H 4
1.3.7.5	2	73	I 6
1.3.7.6	2	73	I 6
1.3.7.7	2	73	I 6
1.3.8	2	73	I 6
1.3.8.1	2	75	K 4
1.3.8.1.1	2	75	K 4
1.3.8.1.2	2	75	K 4
1.3.8.1.3	2	75	K 4
1.3.8.1.4	2	75	K 4
1.3.8.1.5	2	75	K 4
1.3.8.1.5.1	2	76	L 4
1.3.8.2	2	76	L 4
1.3.8.3	2	81	D 7
1.3.8.3.1	2	81	D 7
1.3.8.3.1.1	2	81	D 7
1.3.8.3.1.1.1	2	81	D 7
1.3.8.3.1.1.1.1	2	82	E 7
1.3.8.3.1.1.2	2	84	G 7
1.3.8.3.1.1.3	2	85	H 7
1.3.8.3.1.2	2	85	H 7
1.3.8.3.1.3	2	85	H 7
1.3.8.3.1.3.1	2	86	I 7
1.3.8.3.1.3.2	2	86	I 7
1.3.8.3.1.3.3	2	86	I 7
1.3.8.3.2	2	87	J 7
1.3.8.3.2.1	2	87	J 7
1.3.8.3.2.1.1	2	87	J 7
1.3.8.3.2.1.1.1	2	88	K 7
1.3.8.3.2.1.2	2	88	K 7
1.3.8.3.2.1.3	2	89	L 7
1.3.8.3.2.2	2	89	L 7
1.3.8.3.2.2.1	2	89	L 7
1.3.8.3.2.2.1.1	2	89	L 7
1.3.8.3.2.2.1.2	2	90	M 7
1.3.8.3.2.2.1.3	2	90	M 7
1.3.8.3.2.2.2	2	90	M 7
1.3.8.3.2.2.2.1	2	91	N 7
1.3.8.3.2.2.2.2	2	91	N 7
OTHER SINGLE BUBBLES			
INTERFACIAL DROPS			
INVERSE BUBBLES			
BURNING DROPS			
MULTIPLE BUBBLES			
FEN-BUBBLE PROBLEMS			
MANY-BUBBLE PROBLEMS			
COALESCENCE			
TWO-PHASE FLOW			
FLUIDIZATION			
SOAP BUBBLES			
INVISCID WAVES			
CLASSIFICATION			
NUMBER OF FREE INTERFACES			
SHAPE OF FIXED SURFACES			
TYPE OF WAVE			
INTERFACIAL CONDITIONS			
GOVERNING EQUATIONS			
KORTEWEG-DE VRIES			
OVERALL SUMMARY			
SURFACE WAVES			
INFINITE DEPTH			
PERIODIC PROGRESSING			
GRAVITY			
IRROTATIONAL			
MAXIMUM AMPLITUDE			
ROTATIONAL			
HETEROGENEOUS			
CAPILLARY			
GRAVITY/CAPILLARY			
IRROTATIONAL			
ROTATIONAL			
HETEROGENEOUS			
CNOIDAL AND SOLITARY			
UNIFORM DEPTH			
PERIODIC PROGRESSING			
GRAVITY			
IRROTATIONAL			
MAXIMUM AMPLITUDE			
ROTATIONAL			
HETEROGENEOUS			
CAPILLARY			
GRAVITY/CAPILLARY			
IRROTATIONAL			
ROTATIONAL			
HETEROGENEOUS			
CNOIDAL			
GRAVITY			
IRROTATIONAL			
ROTATIONAL			
HETEROGENEOUS			
GRAVITY/CAPILLARY			
IRROTATIONAL			
ROTATIONAL			

SUBJECT	APPENDIX	PAGE	FICHE
I-3.8.3.2.2.2.3	2	91	N 7
METEROGENEOUS			
SOLITARY	2	91	N 7
GRAVITY	2	91	N 7
IRROTATIONAL	2	91	N 7
MAXIMUM AMPLITUDE	2	92	R 2
ROTATIONAL	2	93	C 8
METEROGENEOUS	2	93	C 8
GRAVITY/CAPILLARY	2	93	C 8
IRROTATIONAL	2	93	C 8
ROTATIONAL	2	93	C 8
METEROGENEOUS	2	94	D 8
PERIODIC PROGRESSING WAVES OVER A WAVY BED	2	94	D 8
GRAVITY	2	94	D 8
GRAVITY/CAPILLARY	2	94	D 8
INTERFACIAL WAVES	2	94	D 8
PERIODIC PROGRESSING	2	95	E 2
CNOIDAL	2	95	E 2
SOLITARY	2	95	E 2
TIDAL WAVES	2	96	F 2
DENSITY FLOWS	2	96	F 2
GRAVITY CURRENTS	2	96	F 2
HORIZONTAL	2	96	F 2
SHALLOW	2	96	F 2
DEEP	2	96	F 2
COLD FRONTS	2	96	F 2
OIL SLICKS	2	97	G 2
SLOPING	2	97	G 2
AVALANCHES	2	97	G 2
LOCK EXCHANGE FLOW	2	97	G 2
SELECTIVE WITHDRAWAL	2	97	G 2
LIQUID FILMS AND SHEETS	2	97	G 2
ONE FREE SURFACE	2	97	G 2
ONE RIGID SURFACE	2	97	G 2
PLANE FLOW	2	97	G 2
AXISYMMETRIC FLOW	2	98	H 2
WITHDRAWAL FROM LIQUIDS	2	98	H 2
DRY SPOTS AND HOT SPOTS	2	99	I 2
RIVULETS	2	99	I 2
SPREADING	2	99	I 2
TWO RIGID SURFACES	2	100	J 2
FLOW BETWEEN ROLLERS	2	100	J 2
TWO FREE SURFACES	2	100	J 2
UNOBSTRUCTED THIN SHEETS	2	100	J 2
WATER BELLS	2	101	K 2
EXTRUSION OF TUBULAR FILMS	2	101	K 2
SURFACE TENSION EFFECTS	2	101	K 2
SOAP BUBBLES	2	101	K 2
MENISCI	2	102	L 2
MISCELLANEOUS	2	102	L 2
LIQUID STORAGE BAGS	2	102	L 2
LIQUID CRYSTALS	2	103	M 2
ROTATING FLOWS	2	103	M 2
ROTATING FLOWS WITH A FREE SURFACE	2	103	M 2
PURE SWIRL	2	103	M 2

SUBJECT	APPENDIX	PAGE	FICHE
I-3.11.1.2			
I-3.11.1.3			
I-3.11.1.4			
I-3.11.1.5			
I-3.11.2			
I-3.11.2.1			
I-3.11.2.2			
I-3.11.3			
I-3.11.3.1			
I-3.11.3.2			
I-3.11.3.3			
I-3.11.4			
I-3.11.5			
I-3.12			
I-3.12.1			
I-3.12.1.1			
I-3.12.1.2			
I-3.12.2			
I-3.13			
I-3.13.1			
I-3.13.1.1			
I-3.13.1.2			
I-3.13.1.3			
I-3.13.2			
I-3.13.2.1			
I-3.13.2.2			
I-3.13.3			
I-3.13.3.1			
I-3.13.3.2			
I-3.13.4			
I-3.13.5			
I-3.13.5.1			
I-3.13.5.2			
I-3.13.6			
I-3.13.6.1			
I-3.13.6.2			
I-3.13.7			
I-3.13.7.1			
I-3.13.7.2			
I-3.13.7.3			
I-3.13.7.4			
I-3.13.7.5			
I-3.13.8			
I-3.13.8.1			
I-3.13.8.2			
I-3.14			
I-3.14.1			
I-3.14.1.1			
I-3.14.1.1.1			
I-3.14.1.1.1.1			
I-3.14.1.1.1.2			
I-3.14.1.1.1.3			
I-3.14.1.1.2			
I-3.14.1.2			
I-3.14.1.2			
FLOW FROM CONTAINERS			
COUETTE FLOW		104	N 8
BETWEEN DISKS		104	N 8
ROTATING JETS		105	B 0
VORTICES		105	B 0
PARALLEL VORTICES		105	B 0
VORTEX RINGS		105	B 0
ROTATING DROPS		106	C 0
NUCLEAR DROP MODEL		106	C 0
LUNAR GLOBULES		107	D 0
BUBBLES IN SPACE		107	D 0
ROTATING MACHINERY		107	D 0
SURFACE WAVES		107	D 0
LUBRICATION CAVITATION		108	E 0
BEARINGS		109	F 0
JOURNAL		110	G 0
THRUST		112	I 0
MECHANICAL SEALS		112	I 0
MISCELLANEOUS PROBLEMS		112	I 0
SELF-SIMILAR FLOWS		112	I 0
ACCELERATED FLOWS		113	J 0
IMPACT OF WEDGES AND CONES		113	J 0
JETS		114	K 0
INVERSE PROBLEMS		114	K 0
WIND TUNNELS		115	L 0
AIR INLETS		115	L 0
METEOROLOGY		115	L 0
CLOUDS		115	L 0
WARM AND COLD FRONTS		115	L 0
SEDIMENT TRANSPORT		116	M 0
NON-NEWTONIAN FLOWS IN PIPES AND CAVITIES		116	M 0
BINGHAM FLUIDS		116	M 0
DILATANT FLUIDS		116	M 0
VORTEX SHEETS		116	M 0
AEROFOILS		117	N 0
SAILS		117	N 0
MATHEMATICAL GENERALIZATIONS		117	N 0
BEURLING		118	P 0
ODHNOFF		118	P 0
BAZALII		118	P 0
MONAMOV		118	P 0
IGLIKOV		118	P 0
FLOWS SUBJECT TO ADDITIONAL CONDITIONS		118	P 0
CONSTANT VELOCITY ON ALL STREAMLINES		118	P 0
CLOSED PARTICLE ORBITS		118	P 0
TRANSONIC FLOWS		119	C 0
UNBOUNDED FLOW PAST OBSTACLES		120	D 0
SUPERCritical FLOWS		120	D 0
SPECIFIC OBSTACLES		121	E 0
CYLINDRICAL OBSTACLES		121	E 0
GENERAL PLANE OBSTACLES		122	F 0
THREE DIMENSIONAL OBSTACLES		122	F 0
DESIGN OF AIRFOILS		122	F 0
DETACHED BOA SHOCK WAVES		122	G 0



SUBJECT	APPENDIX	PAGE	FICHE
I.3.14.1.3			
I.3.14.2			
I.3.14.2.1			
I.3.14.2.2			
I.3.14.2.3			
I.3.14.3			
I.3.14.4			
I.3.15			
I.4			
I.4.0.1			
I.4.0.2			
I.4.0.3			
I.4.1			
I.4.1.1			
I.4.1.2			
I.4.1.3			
I.4.1.3.1			
I.4.1.3.2			
I.4.1.3.3			
I.4.1.4			
I.4.1.5			
I.4.1.6			
I.4.1.7			
I.4.2			
I.4.2.1			
I.4.2.2			
I.4.2.2.1			
I.4.3			
I.4.3.1			
I.4.3.1.1			
I.4.3.1.1.1			
I.4.3.1.1.2			
I.4.3.1.2			
I.4.3.1.3			
I.4.3.1.4			
I.4.3.2			
I.4.3.2.1			
I.4.3.2.2			
I.4.3.2.3			
I.4.3.3			
I.4.3.4			
I.4.3.4.1			
I.4.3.4.1.1			
I.4.3.4.1.1.1			
I.4.3.4.1.1.2			
I.4.3.4.1.2			
I.4.3.4.2			
I.4.4			
I.4.4.1			
I.4.4.1.1			
I.4.4.1.2			
I.4.4.1.3			
I.4.4.2			
I.4.4.2.1			
1.3.14.1.3			
1.3.14.2			
1.3.14.2.1			
1.3.14.2.2			
1.3.14.2.3			
1.3.14.3			
1.3.14.4			
1.3.15			
1.4			
1.4.0.1			
1.4.0.2			
1.4.0.3			
1.4.1			
1.4.1.1			
1.4.1.2			
1.4.1.3			
1.4.1.3.1			
1.4.1.3.2			
1.4.1.3.3			
1.4.1.4			
1.4.1.5			
1.4.1.6			
1.4.1.7			
1.4.2			
1.4.2.1			
1.4.2.2			
1.4.2.2.1			
1.4.3			
1.4.3.1			
1.4.3.1.1			
1.4.3.1.1.1			
1.4.3.1.1.2			
1.4.3.1.2			
1.4.3.1.3			
1.4.3.1.4			
1.4.3.2			
1.4.3.2.1			
1.4.3.2.2			
1.4.3.2.3			
1.4.3.3			
1.4.3.4			
1.4.3.4.1			
1.4.3.4.1.1			
1.4.3.4.1.1.1			
1.4.3.4.1.1.2			
1.4.3.4.1.2			
1.4.3.4.2			
1.4.4			
1.4.4.1			
1.4.4.1.1			
1.4.4.1.2			
1.4.4.1.3			
1.4.4.2			
1.4.4.2.1			
SUBSONIC WAKES			
NOZZLE FLOWS			
TAYLOR			
TRANSITION			
MEYER			
JETS			
WAKES			
FLAMES			
FLOWS THROUGH POROUS MEDIA			
AN EXAMPLE			
TERMINOLOGY			
SUMMARY OF SOME ANALYTICAL AND NUMERICAL METHODS			
THE GOVERNING EQUATIONS			
LINEAR INCOMPRESSIBLE ISOTROPIC HOMOGENEOUS FLOW			
LINEAR COMPRESSIBLE ANISOTROPIC SATURATED FLOW			
NONLINEAR SATURATED FLOW			
LOW SPEED			
HIGH SPEED			
SLIP			
PARTIALLY SATURATED FLOW			
MULTI-PHASE FLOW			
ELECTROKINETIC FLOW			
PLANT ROOTS			
BOUNDARY CONDITIONS			
FIXED BOUNDARIES			
FREE BOUNDARIES			
POINTS OF DETACHMENT			
SEEPAGE			
THROUGH DAMS			
SIMPLE RECTANGULAR DAMS			
THE MODEL PROBLEM			
THE GENERAL CASE			
SIMPLE TRAPEZOIDAL DAMS			
POLYGONAL DAMS			
GENERAL GEOMETRIES			
FROM CANALS, PONDS, AND DITCHES			
SINGLE CHANNEL INTO HALF-PLANE			
SINGLE CHANNEL INTO FINITE LAYER			
MULTIPLE CHANNELS			
DRAINAGE			
WELL FLOW			
SINGLE WELL			
FULLY PENETRATING			
HOMOGENEOUS ISOTROPIC			
OTHER			
PARTIALLY PENETRATING			
MULTIPLE WELLS			
TWO-FLUID FLOWS			
SALT WATER - FRESH WATER INTERFACES			
GYBEN-HERZBERG LENS			
COASTAL AQUIFERS			
LAND RECLAMATION			
OTHER CONFIGURATIONS			
UP-CONING			

SUBJECT	APPENDIX	PAGE	FICHE
I.4.4.2.2	7	23	K 2
I.4.4.3	3	23	K 2
I.4.5	3	23	K 2
I.5	3	24	L 2
I.5.1	3	24	L 2
I.5.1.1	3	24	L 2
I.5.1.1.1	3	24	L 2
I.5.1.1.2	3	24	L 2
I.5.1.2	3	25	M 2
I.5.1.2.1	3	25	M 2
I.5.1.2.2	3	25	M 2
I.5.1.2.3	3	25	M 2
I.5.1.2.4	3	26	N 2
I.5.1.2.4.1	3	26	N 2
I.5.1.2.4.1.1	3	27	U 2
I.5.1.2.4.1.2	3	27	U 2
I.5.1.2.4.1.3	3	27	U 2
I.5.1.2.4.1.4	3	27	U 2
I.5.1.2.4.2	3	27	U 2
I.5.1.2.4.2.1	3	27	U 2
I.5.1.2.4.2.1.1	3	27	U 2
I.5.1.2.4.2.1.2	3	27	U 2
I.5.1.2.4.2.1.3	3	27	U 2
I.5.1.2.4.2.1.4	3	27	U 2
I.5.1.2.4.2.1.5	3	27	U 2
I.5.1.2.4.2.1.6	3	27	U 2
I.5.1.2.4.2.2	3	28	C 2
I.5.1.2.4.2.2.1	3	28	C 2
I.5.1.2.4.3	3	28	C 2
I.5.1.2.4.4	3	28	C 2
I.5.1.2.4.5	3	28	C 2
I.5.1.2.5	3	28	C 2
I.5.1.2.6	3	28	C 2
I.5.1.2.7	3	28	C 2
I.5.1.2.8	3	28	C 2
I.5.2	3	29	D 2
I.5.2.1	3	29	D 2
I.5.2.2	3	29	D 2
I.5.3	3	30	E 2
I.5.3.1	3	31	F 2
I.5.3.1.1	3	31	F 2
I.5.3.1.1.1	3	31	F 2
I.5.3.1.1.1.1	3	31	F 2
I.5.3.1.1.1.2	3	31	F 2
I.5.3.1.1.1.3	3	31	F 2
I.5.3.1.1.1.4	3	31	F 2
I.5.3.1.1.1.5	3	31	F 2
I.5.3.1.1.1.6	3	31	F 2
I.5.3.1.1.2	3	31	F 2
I.5.3.1.1.2.1	3	31	F 2
I.5.3.1.1.2.2	3	31	F 2
I.5.3.1.1.2.3	3	31	F 2
I.5.3.1.1.2	3	31	F 2

SUBJECT	APPENDIX	PAGE	FICHE
I.5.3.1.1.2.1			
I.5.3.1.1.2.2			
I.5.3.1.2			
I.5.3.2			
I.5.3.3			
I.5.3.3.1			
I.5.3.3.1.1			
I.5.3.3.1.1.1			
I.5.3.3.1.1.2			
I.5.3.3.1.1.3			
I.5.3.3.1.2			
I.5.3.3.1.2.1			
I.5.3.3.1.2.2			
I.5.3.3.2			
I.5.3.3.3			
I.5.3.3.4			
I.5.3.3.5			
I.5.3.4			
I.5.3.5			
I.5.3.6			
I.5.3.6.1			
I.5.3.6.1.1			
I.5.3.6.1.2			
I.5.3.6.2			
I.5.3.6.3			
I.5.3.7			
I.5.3.8			
I.5.3.9			
I.5.4			
I.5.4.1			
I.5.4.2			
I.5.4.3			
I.5.5			
I.5.6			
I.5.7			
I.5.8			
I.6			
I.6.1			
I.6.2			
I.6.2.1			
I.6.2.2			
I.6.3			
I.6.4			
I.6.4.1			
I.6.4.2			
I.6.4.3			
I.6.5			
I.6.5.1			
I.6.5.2			
I.6.5.3			
I.6.5.4			
I.6.5.5			
I.7			
I.7.1			
HOLLOW RECTANGLE			
OTHER			
AXISYMMETRIC			
TUBES, CYLINDERS, SPHERES, AND DISKS (RADIAL FORCES)			
PLATES AND BARS (TENSION, COMPRESSION, LOADED)			
NOTCHED (INCLUDING EDGE CRACKS)			
TWO NOTCHES			
V			
SEMI-CIRCULAR			
OTHER			
ONE NOTCH			
V			
OTHER			
INTERNAL CRACKS (THROUGH)			
SURFACE CRACKS			
WITH HOLES			
OTHER GEOMETRIES			
SHELLS			
BEAMS			
INDENTATION AND CONTACT			
FLAT PUNCH			
ON HALF PLANE			
ON FINITE STRIP			
OTHER PUNCH GEOMETRIES			
ROLLING CONTACT			
FINITE DIMENSIONAL STRUCTURES			
EXTRUSION			
GEOMECHANICS			
UNILATERAL PROBLEMS			
SIGNORINI PROBLEMS			
INDENTATION			
INCLUSION			
FINITE DEFORMATIONS			
FRICTION			
OPTIMAL DESIGN			
OTHER PROBLEMS IN THE MECHANICS OF SOLIDS			
HEAT CONDUCTION AND DIFFUSION			
THE GOVERNING EQUATIONS			
BOUNDARY CONDITIONS			
FIXED BOUNDARIES			
FREE BOUNDARIES			
STEADY-STATE PROBLEMS			
STEADILY-MOVING PROBLEMS			
ABLATION			
CONTINUOUS CASTING			
ZONE MELTING			
INTERFACIAL INSTABILITIES			
PLANF INTERFACES			
CYLINDRICAL GROWTH			
DENDRITES			
LAMELLAR EUTECTIC GROWTH			
VAPOR SNAKES			
ELECTROMAGNETICS			
THE GOVERNING EQUATIONS			

SUBJECT	APPENDIX	PAGE	FICHE
I.7.2 BOUNDARY CONDITIONS		60	I 5
I.7.3 DOMAIN FBPS		60	I 5
I.7.3.1 FERROMAGNETISM		60	I 5
I.7.3.1.1 MAGNETIC BUBBLES		60	I 5
I.7.3.2 FERROELECTRICITY		60	I 5
I.7.3.3 SUPERCONDUCTIVITY		60	I 5
I.7.3.3.1 THE JOSEPHSON EFFECT		61	J 5
I.7.3.4 SEMICONDUCTORS		61	J 5
I.7.3.4.1 FIELD EFFECTS		61	J 5
I.7.3.5 ELECTROCHEMISTRY		61	J 5
I.7.4 MAGNETOHYDROSTATICS		61	J 5
I.7.4.1 MAGNETOHYDRODYNAMIC CONTAINMENT		61	J 5
I.7.4.1.1 UNBOUNDED		61	J 5
I.7.4.1.1.1 PLANE		62	K 5
I.7.4.1.1.1.1 CUSPED WITH QUADRIPOLE		62	K 5
I.7.4.1.1.1.2 CUSPED WITH PERIODIC ARRAY OF CONDUCTING COILS		62	K 5
I.7.4.1.1.1.3 PICKET FENCE		62	K 5
I.7.4.1.1.1.4 TOROIDAL		62	K 5
I.7.4.1.1.1.5 BELT PINCH		62	K 5
I.7.4.1.1.2 AXIALLY SYMMETRIC		62	K 5
I.7.4.1.1.2.1 CUSPED WITH QUADRIPOLE		63	L 5
I.7.4.1.1.2.2 TOROIDAL		63	L 5
I.7.4.1.2 BOUNDED		63	L 5
I.7.4.1.2.1 PLANE		63	L 5
I.7.4.1.2.2 CYLINDRICAL		63	L 5
I.7.4.1.2.3 TOROIDAL		63	L 5
I.7.4.1.2.4 HELICAL		64	M 5
I.7.4.1.2.5 THREE DIMENSIONAL		64	M 5
I.7.4.2 THE MAGNETOSPHERE		64	M 5
I.7.4.2.1 CHAPMAN-FERRARO MODEL		64	M 5
I.7.4.2.1.1 PLANE		64	M 5
I.7.4.2.1.1.1 PAST A LINE CURRENT		65	N 5
I.7.4.2.1.1.2 PAST TWO LINE CURRENTS		65	N 5
I.7.4.2.1.1.3 PAST A DIPOLE		65	N 5
I.7.4.2.1.1.4 PAST A DIPOLE WITH NEUTRAL SHEET		65	N 5
I.7.4.2.1.2 THREE-DIMENSIONAL		66	B 5
I.7.4.3 COMETS		66	B 5
I.7.4.4 MAGNETIC STARS		66	B 5
I.7.4.5 PLASMAS WITH VACUUM REGIONS		67	C 5
I.7.5 ELECTROHYDROSTATICS		67	C 5
I.7.5.1 DROPS		67	C 5
I.7.5.1.1 SINGLE		67	C 5
I.7.5.1.2 DOUBLE		67	C 5
I.7.5.2 CONICAL SURFACES		68	D 5
I.7.5.3 FILMS		69	E 5
I.7.5.4 ELECTROCONVECTION		69	E 5
I.7.6 CONTROL SURFACE FBPS		69	E 5
I.7.6.1 ELECTRON GUNS		69	E 5
I.7.6.2 ANTENNAS		69	E 5
I.7.6.3 ELECTROSTATICS		69	E 5
I.7.7 ELECTRIC JETS/ARC JETS		69	E 5
I.8 GRAVITATION		69	E 5
I.8.1 THE GOVERNING EQUATIONS		69	E 5





SUBJECT	APPENDIX	PAGE	FICHE
I-10.2.5			
I-10.3		84	G 7
I-10.4		84	G 7
I-10.5		84	G 7
I-11		85	H 7
I-11.1		85	H 7
I-11.2		85	H 7
I-11.3		85	H 7
I-12		86	I 7
I-12.1		86	I 7
I-12.2		86	I 7
I-12.2.1		86	I 7
I-12.2.2		86	I 7
I-12.2.3		86	I 7
I-12.3		87	J 7
I-12.3.1		87	J 7
I-12.3.2		87	J 7
I-12.4		88	K 7
I-13		88	K 7
I-13.1		88	K 7
I-13.1.1		89	L 7
I-13.1.2		89	L 7
I-13.1.3		89	L 7
I-13.1.4		89	L 7
I-13.1.5		89	L 7
I-13.2		90	M 7
I-13.3		90	M 7
II		90	M 7
II.1		90	M 7
II.1.1		90	M 7
II.1.2		90	M 7
II.1.3		90	M 7
II.1.4		90	M 7
II.1.5		90	M 7
II.1.6		90	M 7
II.2		90	M 7
II.2.1		90	M 7
II.2.1.1		90	M 7
II.2.1.2		90	M 7
II.2.1.2.1		90	M 7
II.2.1.2.2		90	M 7
II.2.1.3		90	M 7
II.2.1.3.1		90	M 7
II.2.1.3.2		90	M 7
II.2.1.4		90	M 7
II.2.1.4.1		90	M 7
II.2.1.4.2		90	M 7
II.2.2		90	M 7
II.2.2.1		90	M 7
II.2.2.1.1		90	M 7
II.2.2.1.2		90	M 7
II.2.2.2		90	M 7
II.2.2.2.1		90	M 7
II.2.2.2.2		90	M 7
THE MINIMAL SURFACE EQUATION			
IMPROPERLY POSED PROBLEMS			
PART II. MATHEMATICAL METHODS			
INTRODUCTION			
THE GENERAL FBP			
WHY FBPS ARE DIFFICULT			
SOME EXPLICIT SOLUTIONS			
APPLICABILITY OF THE VARIOUS MATHEMATICAL METHODS TO FB			
RECENT PROGRESS			
SOME OPEN PROBLEMS			
MATHEMATICAL PRELIMINARIES			
COMPLEX FUNCTION THEORY			
DEFINITIONS			
CONFORMAL MAPPING			
THE SCHWARZ-CHRISTOFFEL MAPPING			
VARIATIONAL PRINCIPLES			
BEHAVIOR OF ANALYTIC FUNCTIONS			
NEAR THE BOUNDARY			
NEAR SINGULARITIES			
BOUNDARY VALUE PROBLEMS			
RIEMANN-HILBERT			
RIEMANN-HILBERT-POINCARÉ			
REPRESENTATION OF SOLUTIONS OF DIFFERENTIAL EQUATIONS			
COMPLEX FUNCTIONS			
LAPLACE EQUATION			
BINHARMONIC EQUATION			
GENERALIZATIONS OF COMPLEX FUNCTION THEORY			
GENERALIZED ANALYTIC FUNCTIONS/PSEUDOANALYTIC FUNCTIONS			
BERGMAN/VEKUA/GILEAD REPRESENTATIONS			

SUBJECT	APPENDIX	PAGE	FICHE
II.2.2.3		9	J 1
II.2.2.3.1		9	J 1
II.2.2.3.2		10	K 1
II.2.2.3.2.1		10	K 1
II.2.2.3.2.1.1		10	K 1
II.2.2.3.2.1.2		10	K 1
II.2.2.3.2.2		11	L 1
II.2.2.3.2.2.1		11	L 1
II.2.2.3.2.2.2		12	M 1
II.2.2.3.2.2.3		12	M 1
II.2.2.3.2.2.4		12	M 1
II.2.2.3.3		12	M 1
II.2.2.3.3.1		12	M 1
II.2.2.3.3.2		12	M 1
II.2.2.3.3.3		12	M 1
II.2.3		12	M 1
II.2.3.1		13	N 1
II.2.3.1.1		13	N 1
II.2.3.1.1.1		13	N 1
II.2.3.1.1.1.1		14	B 2
II.2.3.1.1.1.2		14	B 2
II.2.3.1.1.1.3		14	B 2
II.2.3.1.1.2		15	C 2
II.2.3.1.1.2.1		15	C 2
II.2.3.1.2		15	C 2
II.2.3.1.2.1		15	C 2
II.2.3.1.2.2		15	C 2
II.2.3.2		15	C 2
II.2.3.2.1		15	C 2
II.2.3.2.2		15	C 2
II.2.3.3		15	C 2
II.2.4		16	D 2
II.2.5		17	E 2
II.3		17	E 2
II.3.1		18	F 2
II.3.2		18	F 2
II.3.3		19	G 2
II.3.3.1		19	G 2
II.3.3.2		19	G 2
II.3.3.3		20	H 2
II.3.3.4		20	H 2
II.3.3.5		20	H 2
II.3.3.6		20	H 2
II.3.4		20	H 2
II.3.4.1		20	H 2
II.3.4.2		20	H 2
II.3.4.2.1		20	H 2
II.3.4.2.1.1		20	H 2
II.3.4.2.1.1.1		21	I 2
II.3.4.2.1.1.2		21	I 2
II.3.4.2.1.2		21	I 2
II.3.4.2.1.3		22	J 2
II.3.4.2.1.4		22	J 2
II.3.4.2.1.5		22	J 2
INTEGRAL REPRESENTATIONS			
SINGULAR INTEGRALS			
SURFACE DISTRIBUTIONS			
PLANE			
SINGLE LAYER			
DOUBLE LAYER			
AXISYMMETRIC			
GREEN'S REPRESENTATION			
SOURCE RINGS			
VORTEX RINGS			
AXIAL DISTRIBUTIONS			
OTHER REPRESENTATIONS			
MARTENSEN			
GIRAUD			
GILBERT			
THEORY OF ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS			
SECOND ORDER EQUATIONS			
TWO VARIABLES			
LINEAR			
LAPLACE			
UNIFORMLY ELLIPTIC			
DEGENERATE			
NONLINEAR			
IN DIVERGENCE FORM			
MORE THAN TWO VARIABLES			
LINEAR			
NONLINEAR			
HIGHER ORDER EQUATIONS			
BIMHARMONIC			
NAVIER STOKES			
BEHAVIOUR AT CORNERS AND NEAR SINGULARITIES			
SOME DIFFERENTIAL GEOMETRY			
SOBOLEV SPACES AND RELATED SPACES			
REVIEW OF EXISTENCE AND UNIQUENESS PROOFS			
THE NEED FOR EXISTENCE AND UNIQUENESS PROOFS			
EXAMPLES OF NON-EXISTENCE			
EXISTENCE PROOFS			
DIRECT CONSTRUCTION			
INTEGRAL EQUATIONS			
NONLINEAR BOUNDARY VALUE PROBLEMS			
CONTINUATION METHODS			
CLASSICAL VARIATIONAL METHODS			
VARIATIONAL INEQUALITIES			
EXAMPLES OF NON-UNIQUENESS			
PHYSICAL EXAMPLES			
MATHEMATICAL EXAMPLES			
FLUID MECHANICS			
CAVITY FLOWS			
PAST A PLATE			
GENERAL OBSTACLES			
JETS			
ONE UNOBTSTRUCTED SURFACE			
ONE OBSTRUCTED SURFACE			
BUBBLES			

SUBJECT	APPENDIX	PAGE	FICHE
II-3.4.2.1.6			
II-3.4.2.2			
II-3.4.2.3			
II-3.4.2.4			
II-3.5			
II-3.5.1			
II-3.5.1.1			
II-3.5.1.2			
II-3.5.2			
II-3.5.3			
II-3.5.4			
II-4			
II-4.1			
II-4.1.1			
II-4.1.1.1			
II-4.1.1.2			
II-4.1.1.3			
II-4.1.1.4			
II-4.1.1.5			
II-4.1.2			
II-4.1.2.1			
II-4.1.2.1			
II-4.1.3			
II-4.1.4			
II-4.2			
II-4.2.1			
II-4.2.2			
II-4.2.3			
II-4.2.4			
II-4.2.5			
II-4.2.6			
II-4.3			
II-4.4			
II-4.5			
II-4.6			
II-5			
II-5.1			
II-5.2			
II-5.2.1			
II-5.2.2			
II-5.2.3			
II-5.3			
II-5.3.1			
II-5.3.2			
II-5.3.3			
II-5.4			
II-5.4.1			
II-5.4.2			
II-5.4.3			
II-5.5			
II-5.5.1			
II-5.5.2			
II-5.6			
II-6			
WAVES	4	23	K 2
POROUS FLOW	4	23	K 2
MECHANICS OF SOLIDS	4	23	K 2
OTHER PROBLEMS	4	24	L 2
UNIQUENESS PROOFS	4	24	L 2
MAXIMUM PRINCIPLE	4	24	L 2
COMPARISON METHODS ON UNKNOWN DOMAINS	4	25	M 2
NONLINEAR EQUATIONS ON KNOWN DOMAINS	4	25	M 2
METHOD OF CONTINUITY	4	25	M 2
VARIATIONAL METHODS	4	25	M 2
VARIATIONAL INEQUALITIES	4	26	N 2
MAPPING THE FREE DOMAIN ONTO A KNOWN DOMAIN	4	26	N 2
THE PHI-PSI MAPPING AND VARIANTS	4	26	N 2
LAPLACE'S EQUATION IN THE PLANE	4	26	N 2
DEPENDENT VARIABLES X AND Y	4	26	N 2
DEPENDENT VARIABLE LN(Q) AND THETA	4	27	B 3
DEPENDENT VARIABLE $z = x + i \cdot y$	4	28	C 3
THE ZHUKOVSKII MAPPING	4	28	C 3
THE HAMEL MAPPING	4	28	C 3
EQUATIONS IN DIVERGENCE FORM	4	28	C 3
AXIALLY SYMMETRIC PROBLEMS	4	29	D 3
OTHER PROBLEMS	4	29	D 3
THREE-DIMENSIONAL PROBLEMS	4	30	E 3
MODIFIED PHI-PSI MAPPINGS	4	30	E 3
THE HODOGRAPH MAPPING AND VARIANTS	4	30	E 3
U*IV	4	30	E 3
$u \cdot \exp(-i \cdot \theta)$	4	30	E 3
$\ln(1/Q) + i \cdot \theta$	4	30	E 3
SUBSONIC COMPRESSIBLE FLOW	4	30	E 3
TRANSONIC FLOW	4	31	F 3
AXIALLY SYMMETRIC	4	31	F 3
INTEGRAL TRANSFORMS	4	31	F 3
THE VON MISES MAPPING ( $\lambda, \psi$ )	4	32	G 3
CONFORMAL MAPPING ONTO A KNOWN DOMAIN	4	32	G 3
BRUTE FORCE MAPPINGS	4	33	H 3
THE HODOGRAPH METHOD	4	33	H 3
GENERAL DESCRIPTION	4	34	I 3
CONSTRUCTION OF MAPPINGS	4	34	I 3
SCHWARZ-CHRISTOFFEL	4	35	J 3
RIEMANN-HILBERT	4	35	J 3
NUMERICAL METHODS	4	35	J 3
EXAMPLES	4	36	K 3
THE KIRCHHOFF-RAYLEIGH CAVITY	4	36	K 3
MAGNETOHYDRODYNAMIC EQUILIBRIUM	4	36	K 3
POROUS FLOW. AN EXAMPLE OF THE USE OF RIEMANN SURFACES	4	36	K 3
FINAL COMPUTATIONS AND VERIFICATION	4	36	K 3
THE PARAPETER PROBLEMS	4	36	K 3
PHYSICALLY INVALID SOLUTIONS	4	37	L 3
THEORY OF SIMPLE FLOWS	4	37	L 3
MODIFICATIONS OF THE HODOGRAPH METHOD	4	37	L 3
METHODS OF CHAPLYGIN, NICOLAU, AND FALKOVICH	4	37	L 3
METHOD OF MACKEY	4	38	M 3
SOME POSSIBLE NEW APPLICATIONS	4	38	M 3
ONE DIMENSIONAL INTEGRAL AND INTEGRODIFFERENTIAL EQUATIONS	4	38	M 3



SUBJECT	APPENDIX	PAGE	FICHE
II.6.1		39	N 3
II.6.2		39	N 3
II.6.3		40	H 4
II.6.3.1		40	H 4
II.6.3.1.1		40	B 4
II.6.3.1.1.1		42	B 4
II.6.3.1.1.2		42	D 4
II.6.3.2		42	D 4
II.6.3.2.1		42	D 4
II.6.3.2.2		44	F 4
II.6.3.2.3		44	F 4
II.6.3.2.4		45	G 4
II.6.3.2.5		47	I 4
II.6.3.2.6		47	I 4
II.6.3.2.7		47	I 4
II.6.3.3		48	J 4
II.6.3.3.1		48	J 4
II.6.3.3.2		49	K 4
II.6.4		49	K 4
II.6.4.1		49	K 4
II.6.4.1.1		50	L 4
II.6.4.1.2		50	L 4
II.6.4.1.3		51	M 4
II.6.4.1.4		52	M 4
II.6.4.2		52	M 4
II.6.4.2.1		52	N 4
II.6.4.2.2		54	C 5
II.6.4.2.3		55	D 5
II.6.4.2.4		56	E 5
II.6.4.2.5		57	F 5
II.6.4.2.6		57	F 5
II.6.5		57	F 5
II.7		57	F 5
II.7.1		57	F 5
II.7.2		58	G 5
II.7.2.1		58	G 5
II.7.2.2		58	G 5
II.7.2.3		58	G 5
II.7.3		59	H 5
II.7.3.1		60	I 5
II.7.3.2		61	J 5
II.7.3.2.1		62	K 5
II.7.3.2.2		62	K 5
II.7.3.2.3		63	L 5
II.7.3.2.4		63	L 5
II.7.3.3		64	M 5
II.8			
II.8.1			
II.8.1.1			
II.8.1.2			
II.8.1.3			
II.8.1.4			
II.8.1.5			
II.8.2			

INTRODUCTION	4	
SOME IMPORTANT KERNELS AND MAPPINGS	4	
CLASSIFICATION OF EQUATIONS ARISING FROM FPPS	4	
NONLINEAR INTEGRAL EQUATIONS	4	
HAMMERSTEIN	4	
LIAPUNOV-SCHWIDT	4	
URYSONN	4	
FUNCTIONAL INTEGRAL EQUATIONS	4	
NEKRASOV-MILNE-THOMPSON	4	
VILLAT(ARC LENGTH)	4	
BIRKHOFF-ZARANTONELLO(CURVATURE)	4	
GRAVITY FLOWS	4	
SEDOV	4	
WOODS	4	
MISCELLANEOUS	4	
INTEGRODIFFERENTIAL EQUATIONS	4	
LICHTENSTEIN	4	
MISCELLANEOUS EQUATIONS	4	
EXISTENCE PROOFS	4	
CONSTRUCTIVE	4	
MAJORANT SERIES	4	
CONTRACTION MAPPING	4	
MONOTONE OPERATORS	4	
HAMMERSTEIN EQUATIONS	4	
NON-CONSTRUCTIVE	4	
FIXED POINT THEOREM OF SCHAUDER	4	
LERAY-SCHAUDER THEORY	4	
BIFURCATION THEORY	4	
THEORY OF NONLINEAR POSITIVE OPERATORS	4	
IMPLICIT FUNCTION THEORY AND PARAMETER CONTINUATION	4	
VARIATIONAL METHODS	4	
UNIQUENESS PROOFS	4	
OTHER FUNCTIONAL EQUATIONS	4	
DIFFERENCE-DIFFERENTIAL	4	
DIFFERENTIAL EQUATIONS	4	
BAZALII'S EQUATION	4	
LEWY'S EQUATION	4	
RAPOPORT'S EQUATION	4	
INTEGRAL EQUATIONS ON UNKNOWN DOMAINS	4	
TREFTZ (GREEN'S IDENTITY)	4	
SURFACE DISTRIBUTIONS	4	
PLANE	4	
RING SOURCES AND VORTICES	4	
SELFCONSISTENT METHOD AND MOMENT METHOD	4	
AXIAL	4	
BROSONSKI AND DE VRIES	4	
NONLINEAR BOUNDARY VALUE PROBLEMS	4	
SERIES EXPANSIONS	4	
LEVI-CIVITA'S EXPANSION	4	
FRIEDRICH'S EXPANSION	4	
GOUYON'S EXPANSION	4	
SCHAUDER ESTIMATES	4	
DOMAIN PERTURBATION	4	
SUCCESSIVE APPROXIMATION	4	

SUBJECT	APPENDIX	PAGE	FIGURE
II-8.3			
II-8.4			
II-8.5			
II-8.5.1			
II-8.5.2			
II-8.6			
II-9			
II-9.1			
II-9.2			
II-9.3			
II-9.4			
II-9.5			
II-9.6			
II-10			
II-10.1			
II-10.2			
II-10.2.1			
II-10.2.2			
II-10.2.2.1			
II-10.2.2.2			
II-10.2.2.3			
II-10.2.2.4			
II-10.3			
II-10.3.1			
II-10.3.2			
II-10.3.2.1			
II-10.3.2.2			
II-10.3.2.3			
II-10.3.2.4			
II-10.3.2.5			
II-10.3.2.6			
II-10.3.3			
II-10.3.3.1			
II-10.3.3.2			
II-10.3.3.3			
II-10.3.3.4			
II-10.3.3.5			
II-10.3.3.6			
II-10.3.4			
II-10.3.5			
II-10.3.6			
II-10.3.7			
II-10.4			
II-10.4.1			
II-10.4.2			
II-10.4.3			
II-10.4.4			
II-10.5			
II-10.5.1			
II-10.5.2			
II-10.5.3			
II-10.5.4			
II-10.5.5			
II-10.5.6			
ASYMPTOTIC EXPANSIONS		64	M 5
NONLINEAR STEKLOV PROBLEMS		65	N 5
"DEGENERATE PROBLEMS"		66	B 4
THE MINIMAL SURFACE EQUATION		66	B 6
OTHER DEGENERATE PROBLEMS		66	B 6
EQUATIONS OF MIXED TYPE		66	B 6
CONTINUATION METHODS		66	B 4
GENERAL DESCRIPTION		66	B 6
INCREMENTAL METHODS		66	B 6
TIME-DEPENDENT METHODS		67	C 6
DIMENSIONAL PERTURBATION (GARAFERIAN)		68	D 6
AN ACROSS THE FREE BOUNDARY ANALYTIC CONTINUATION		68	D 6
ANALYTIC CONTINUATION INTO THE COMPLEX PLANE		69	E 6
CLASSICAL VARIATIONAL METHODS		69	E 6
INTRODUCTION		70	F 4
COMPUTATION OF VARIATIONS		70	F 4
FIXED DOMAINS		70	F 6
VARIABLE DOMAINS		70	F 6
HADAMARD'S FORMULA FOR THE GREEN'S FUNCTION		71	G 4
AREA		72	H 4
VIRTUAL MASS		72	H 4
EIGENVALUES		72	H 4
FORMULATION OF VARIATIONAL PROBLEMS		72	H 4
GENERAL REMARKS		72	H 4
PRINCIPLE OF MINIMUM ENERGY		73	I 4
ROTATING DROPS		73	I 4
ELECTROSTATIC DROPS		73	I 4
ZONE MELTING		73	I 4
BUBBLES		73	I 4
LIQUID CRYSTALS		73	I 4
NONLINEAR ELASTICITY		74	J 4
HAMILTON'S VARIATIONAL PRINCIPLE		74	J 4
CAVITIES		74	J 4
JETS		74	J 4
WAVES		74	J 4
VORTEX SHEETS		75	K 4
SUBSONIC AND TRANSONIC FLOWS		75	K 4
ROTATING FLUIDS		75	K 4
DISSIPATIVE SYSTEMS		76	L 4
DUAL(COMPLEMENTARY) VARIATIONAL PRINCIPLES		76	L 4
MINIMAX PRINCIPLES		77	M 4
MISCELLANEOUS VARIATIONAL PRINCIPLES		77	M 4
EXISTENCE AND UNIQUENESS PROOFS		78	N 4
CAVITIES		78	N 4
VORTEX SHEETS		78	N 4
JETS		78	N 4
MAGNETOHYDROSTATIC CONTAINMENT		78	N 4
GENERALIZATIONS		78	N 4
NONEXISTENCE OF VARIATIONAL PRINCIPLES		79	P 4
MINIMAL SURFACES(PARAMETRIC FORMULATION)		79	P 4
VORFOLDS AND GEOMETRIC MEASURE THEORY		80	C 7
VARIATIONAL INEQUALITIES		81	D 7
CRITICAL POINT THEORY		81	D 7
KUDRJACEV		81	D 7

SUBJECT	APPENDIX	PAGE	PICME
II-10.6		81	D 7
II-11		82	E 7
II-11.1		82	E 7
II-11.2		84	G 7
II-11.2.1		84	G 7
II-11.2.2		84	G 7
II-11.3		85	H 7
II-11.3.1		85	H 7
II-11.3.2		85	H 7
II-11.3.3		86	I 7
II-11.3.3.1		86	I 7
II-11.3.3.1.1		86	I 7
II-11.3.3.1.2		88	K 7
II-11.3.3.2		88	K 7
II-11.4		88	K 7
II-11.4.1		89	L 7
II-11.4.2		89	L 7
II-11.4.3		90	M 7
II-11.4.4		91	N 7
II-11.4.5		92	P 7
II-11.4.6		92	P 7
II-11.4.7		92	P 7
II-11.4.8		93	C 7
II-11.4.9		93	C 7
II-11.4.10		93	C 7
II-11.5		93	C 7
II-11.5.1		93	C 7
II-11.5.1.1		94	D 7
II-11.5.1.2		94	D 7
II-11.5.1.2.1		94	D 7
II-11.5.1.2.2		95	E 7
II-11.5.1.3		95	E 7
II-11.5.2		96	F 7
II-11.5.3		96	F 7
II-11.5.3.1		97	G 7
II-11.5.3.2		97	G 7
II-11.5.4		98	H 7
II-11.5.5		98	H 7
II-11.6		98	H 7
II-11.7		99	I 7
II-11.7.1		99	I 7
II-11.7.1.1		99	I 7
II-11.7.1.2		99	I 7
II-11.7.1.3		100	J 7
II-11.7.1.4		100	J 7
II-11.7.2		100	J 7
II-11.7.2.1		101	K 7
II-11.7.2.2		101	K 7
II-11.7.2.3		102	L 7
II-11.7.2.3.1		102	L 7
II-11.7.2.3.2		103	M 7
VARIATIONAL METHODS FOR INTEGRAL EQUATIONS			
VARIATIONAL INEQUALITIES			
HISTORY OF INEQUALITIES IN MECHANICS			
EXPLANATORY EXAMPLES			
THE SIGNORINI PROBLEM FOR A MEMBRANE			
ONE-DIMENSIONAL PROBLEMS			
BASIC THEORY OF VARIATIONAL INEQUALITIES			
COERCIVE			
SEMI-COERCIVE			
NON-COERCIVE			
LINEAR VARIATIONAL INEQUALITIES			
NONLINEAR VARIATIONAL INEQUALITIES			
SOME PROPERTIES OF SOBOLEV SPACES			
INEQUALITIES IN H <sup>1</sup>			
TYPICAL CONVEX SETS K			
COINCIDENCE SETS I(CU)			
EXTENSIONS OF FUNCTIONS			
OTHER FORMULATIONS			
UNILATERAL MINIMIZATION			
COMPLEMENTARITY PROBLEMS			
DUALITY			
NONLINEAR EQUATIONS			
SUBDIFFERENTIALS			
LINEAR RESTRAINTS			
FIXED POINT(PROJECTIONS)			
EPIGRAPH			
PENALIZATION AND REGULARIZATION			
LINEAR OPTIMIZATION			
PROPERTIES OF SOLUTIONS OF VARIATIONAL INEQUALITIES			
THE LAPLACE OPERATOR			
REGULARITY			
EQUIVALENT PROBLEMS			
DIFFERENT CHOICES OF THE CONVEX SET K			
SUBHARMONIC PROBLEMS			
PROPERTIES OF THE FREE BOUNDARY AND COINCIDENCE SET			
LINEAR SECOND ORDER EQUATIONS			
NONLINEAR SECOND ORDER EQUATIONS			
THE MINIMAL SURFACE EQUATION			
EQUATIONS IN DIVERGENCE FORM			
THE BIHARMONIC EQUATION			
OTHER PROBLEMS			
QUASI-VARIATIONAL INEQUALITIES			
APPLICATIONS			
METHODS OF FORMULATION			
UNILATERAL MINIMIZATION			
COMPLEMENTARITY			
BALOCCHI'S TRANSFORMATION			
QUASI-VARIATIONAL INEQUALITIES			
FLUID MECHANICS			
CUSPED WAKES			
LUBRICATION CAVITATION			
NON-NEWTONIAN FLOWS			
BINGHAM FLUIDS			
DILATANT FLUIDS			



SUBJECT	APPENDIX	PAGE	FIGURE
II.11.7.2.4		103	M 8
II.11.7.3		103	M 8
II.11.7.3.1		104	N 8
II.11.7.3.1.1		104	N 8
II.11.7.3.1.1.1		104	N 8
II.11.7.3.1.1.2		105	B 9
II.11.7.3.1.2		105	B 9
II.11.7.3.1.3		106	C 9
II.11.7.3.1.4		106	C 9
II.11.7.3.2		106	C 9
II.11.7.3.3		106	C 9
II.11.7.3.4		107	D 9
II.11.7.3.5		107	D 9
II.11.7.4		107	D 9
II.11.7.4.1		107	D 9
II.11.7.4.1.1		108	E 9
II.11.7.4.1.2		109	F 9
II.11.7.4.1.3		109	F 9
II.11.7.4.1.4		110	G 9
II.11.7.4.1.5		110	G 9
II.11.7.4.2		111	H 9
II.11.7.4.3		111	H 9
II.11.7.4.4		112	I 9
II.11.7.5		112	I 9
II.11.7.6		112	I 9
II.11.7.7		112	I 9
II.11.7.7.1		112	I 9
II.11.7.8		112	I 9
II.11.7.9		112	I 9
II.12		112	I 9
II.12.1		112	I 9
II.12.2		113	J 9
II.12.2.1		113	J 9
II.12.2.2		113	J 9
II.12.2.3		113	J 9
II.12.2.4		114	K 9
II.12.2.5		114	K 9
II.12.2.6		115	L 9
II.12.3		115	L 9
II.12.4		115	L 9
II.12.5		115	L 9
II.13		116	M 9
II.13.1		116	M 9
II.13.2		117	N 9
II.13.3		117	N 9
II.13.4		119	O 10
II.13.5		119	O 10
II.13.5.1		119	O 10
II.13.5.2		119	O 10
II.13.5.3		119	O 10
II.13.5.4		120	P 10
II.14		120	P 10
II.14.1		120	P 10
II.14.1.1		120	P 10



SUBJECT	APPENDIX	PAGE	FICHE
II.14.1.2		121	E10
II.14.1.3		121	E10
II.14.1.3.1		122	F10
II.14.1.3.1.1		122	F10
II.14.1.3.1.2		122	F10
II.14.1.3.1.2.1		122	F10
II.14.1.3.2		122	F10
II.14.1.3.3		122	G10
II.14.1.3.4		124	M10
II.14.1.3.5		124	M10
II.14.2		124	M10
II.14.2.1		124	M10
II.14.2.1.1		124	M10
II.14.2.1.1.1		125	I10
II.14.2.1.1.2		125	I10
II.14.2.1.2		126	J10
II.14.2.1.3		127	K10
II.14.2.2		127	K10
II.14.3		127	K10
II.14.3.1		127	K10
II.14.3.2		128	L10
II.14.3.3		128	L10
II.14.4		128	L10
II.14.5		128	L10
II.14.6		128	L10
II.15		128	L10
II.15.1		129	M10
II.15.2		129	M10
II.15.2.1		129	M10
II.15.2.2		129	M10
II.15.2.3		130	N10
II.15.3		130	N10
II.15.4		130	N10
II.15.5		130	N10
III		1	P1
III.1		1	P1
III.1.1		1	B1
III.1.2		1	B1
III.1.2.1		1	B1
III.1.2.2		4	E1
III.1.2.3		4	F1
III.1.2.4		5	F1
III.1.2.5		5	F1
III.1.2.6		6	G1
III.1.2.7		7	H1
III.1.3		7	H1
III.1.4		9	J1
III.2		9	J1
III.2.1		9	J1
III.2.2		10	K1
III.2.3		12	M1
III.2.4		15	C2
III.2.5		17	E2
III.2.6		17	E2

SUBJECT	APPENDIX	PAGE	FIGURE
III.2.7			
III.2.8			
III.2.9			
III.2.10			
III.2.11			
III.3			
III.3.0			
III.3.1			
III.3.2			
III.3.2.1			
III.3.2.2			
III.3.2.3			
III.3.2.4			
III.3.2.6.1			
III.3.2.6.2			
III.3.2.6.3			
III.3.2.5			
III.3.2.5.1			
III.3.2.5.2			
III.3.2.5.3			
III.3.2.5.4			
III.3.2.6			
III.3.3			
III.3.3.1			
III.3.3.1.1			
III.3.3.1.2			
III.3.3.2			
III.3.3.2.1			
III.3.3.2.2			
III.3.3.2.3			
III.3.3.2.4			
III.3.3.3			
III.3.3.4			
III.3.4			
III.3.4.1			
III.3.4.2			
III.4			
III.4.1			
III.4.2			
III.4.3			
III.4.3.1			
III.4.3.2			
III.4.3.3			
III.4.3.3.1			
III.4.3.3.2			
III.4.3.3.3			
III.4.3.3.4			
III.4.3.4			
III.4.4			
III.4.4.1			
III.4.4.2			
III.4.4.3			
III.4.5			
III.4.6			
ANALOG METHODS			
SOLUTION OF LINEAR ALGEBRAIC EQUATIONS			
SOLUTION OF NONLINEAR ALGEBRAIC EQUATIONS			
TREATMENT OF SINGULARITIES			
COMPARISON OF METHODS			
TRIAL FREE BOUNDARY METHODS			
INTRODUCTION			
GENERAL DESCRIPTION			
COMPUTATION OF THE TRIAL SOLUTION			
FINITE DIFFERENCES			
FINITE ELEMENTS			
GALERKIN METHODS			
INTEGRAL EQUATIONS AND SURFACE DISTRIBUTIONS			
TREFFTZ			
SURFACE DISTRIBUTIONS			
SELF-CONSISTENT METHOD AND THE MOMENT METHOD			
ANALOG METHODS			
ELECTROLYTIC TANK			
THE HELE-SHAW ANALOG			
RESISTANCE NETWORKS			
CONDUCTING PAPER AND CARBON			
GRAPHICAL METHODS			
MOVEMENT OF THE BOUNDARY			
CHOICE OF BOUNDARY CONDITIONS			
GARABEDIAN'S METHOD			
INTEGRAL RELATIONS			
MOVEMENT STRATEGY			
LOCAL METHODS			
INTEGRAL METHODS			
GLOBAL METHODS			
ESTIMATION OF AN UNKNOWN CONSTANT			
A CONVERGENCE PROOF FOR A MODEL PROBLEM			
SUMMARY OF NUMERICAL EXPERIENCE			
CONCLUDING REMARKS			
ERROR ESTIMATES			
TREATMENT OF SINGULARITIES			
CONTINUATION METHODS			
GENERAL DESCRIPTION			
INCREMENTAL METHODS IN ELASTICITY AND PLASTICITY			
TIME-DEPENDENT METHODS			
PARTICLE-IN-CELL AND MARKER-AND-CELL			
SHOCK CAPTURE			
POROUS FLOW PROBLEMS			
CONSTRUCTION OF FINITE ELEMENT METHODS			
SATURATED FLOW			
SATURATED-UNSATURATED FLOW			
MULTIPHASE AND COUPLED FLOW			
ARTIFICIAL TIME-DEPENDENCE			
CONTINUATION ACROSS BOUNDARIES			
CONTINUATION INTO THE COMPLEX PLANE			
DOMAIN ENLARGEMENT			
ANALYTIC CONTINUATION			
DIMENSIONAL PERTURBATION			
INVARIANT EMBEDDING			

SUBJECT	APPENDIX	PAGE	FICHE
III-5			
III-5.1		70	F 6
III-5.2		70	F 6
III-5.2.1		71	G 6
III-5.2.1.1		71	G 6
III-5.2.1.2		72	H 6
III-5.2.1.2.1		72	H 6
III-5.2.1.2.2		73	I 6
III-5.2.1.3		73	I 6
III-5.2.2		73	I 6
III-5.2.2.1		74	J 6
III-5.2.2.2		74	J 6
III-5.2.2.3		74	J 6
III-5.2.2.3.1		75	K 6
III-5.2.2.4		75	K 6
III-5.2.2.5		75	K 6
III-5.3		75	K 6
III-5.3.1		75	K 6
III-5.3.2		76	L 6
III-5.4		76	L 6
III-5.5		76	L 6
III-5.6		76	L 6
III-5.6.1		76	L 6
III-5.6.2		77	M 6
III-5.7		77	M 6
III-5.8		78	N 6
III-5.9		78	N 6
III-5.9.1		78	N 6
III-5.9.2		78	N 6
III-5.10		78	N 6
III-6		79	B 7
III-6.1		79	B 7
III-6.2		80	C 7
III-6.2.1		81	D 7
III-6.2.2		81	D 7
III-6.3		82	E 7
III-6.3.1		82	E 7
III-6.3.2		83	F 7
III-6.3.2.1		83	F 7
III-6.3.2.2		84	G 7
III-6.3.3		84	G 7
III-6.3.4		85	H 7
III-6.3.5		85	H 7
III-6.4		86	I 7
III-6.4.1		86	I 7
III-6.5		86	I 7
III-6.6		86	I 7
III-7		86	I 7
III-7.1		87	J 7
III-7.2		87	J 7
III-7.2.1		87	J 7
III-7.2.2		88	K 7
III-7.2.3		88	K 7
III-7.2.3.1		88	K 7



SUBJECT	APPENDIX	PAGE	FIGURE
III.7.2.3.2			
III.8			
III.8.1			
III.8.2			
III.8.2.1			
III.8.2.2			
III.8.2.3			
III.8.2.4			
III.8.2.5			
III.8.2.6			
III.8.2.7			
III.8.3			
III.8.3.1			
III.8.3.2			
III.8.3.3			
III.8.3.4			
III.8.3.5			
III.8.4			
III.A			
ELECTROSTATIC DROPS	5	89	L 7
VARIATIONAL INEQUALITIES	5	89	L 7
GENERAL DESCRIPTION	5	89	L 7
METHODS OF APPROXIMATION	5	89	L 7
MINIMIZATION	5	90	M 7
COMPLEMENTARITY	5	91	N 7
DUALITY	5	92	B 8
NONLINEAR EQUATIONS	5	92	B 8
FIXED POINTS(PROJECTION)	5	93	C 8
PENALIZATION AND REGULARIZATION	5	93	C 8
FINITE DIMENSIONAL APPROXIMATION OF VARIATIONAL INEQUAL	5	93	C 8
SOLUTION OF FINITE DIMENSIONAL VARIATIONAL INEQUALITIES	5	94	D 8
RELAXATION WITH MINIMIZATION	5	94	D 8
METHODS FOR COMPLEMENTARITY PROBLEMS	5	97	G 8
DUALITY	5	99	I 8
LINEAR PROGRAMMING	5	99	I 8
OTHER METHODS	5	99	I 8
QUASI-VARIATIONAL INEQUALITIES	5	100	J 8
PUBLISHED PROGRAMS	5	100	J 8



REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER #1793	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER 9	Technical summary rept.
4. TITLE (and Subtitle) A BIBLIOGRAPHY OF FREE BOUNDARY PROBLEMS.		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period	
		6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s) Colin W./Cryer		8. CONTRACT OR GRANT NUMBER(s) 15	DAAG29-75-C-0024 DCR75-03838 NSF-DCR 75-03838
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 3- Applications of Mathematics	
11. CONTROLLING OFFICE NAME AND ADDRESS See Item 18 below		12. REPORT DATE Sep 77	11 12/36p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES 33	
		15. SECURITY CLASS. (of this report) UNCLASSIFIED	
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES U.S. Army Research Office P. O. Box 12211 Research Triangle Park North Carolina 27709 National Science Foundation Washington, D.C. 20550			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Free boundary problems, Fluid mechanics, Porous flow, Mechanics of solids, Heat conduction and diffusion, Electromagnetism, Gravitation, Mathematical methods, Numerical methods.			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A free boundary problem is a (steady-state) boundary value problem involving differential equations on domains parts of whose boundaries, the free boundaries, are unknown and must be determined as part of the solution. Free boundary problems arise in all branches of continuum mechanics. The bibliography contains about 3300 references most of which are classified according to one or more of approximately 1200 subjects. Listings by author and by subject are given in appendices; these are on microfiche, but paper copies can be obtained by writing to the Mathematics Research Center.			